

More Thoughts on the Quantum Theory of Stable de Sitter Space

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ABSTRACT: I review and update ideas about the quantum theory of de Sitter space. New results include a quantum relation between energy and entropy of states in the causal patch, which is satisfied by small dS black holes. I also discuss the preliminaries of a quantum theory in global coordinates, which is invariant under a q-deformed version of the de Sitter supergroup. In this context I outline an algebraic derivation of the CSB scaling relation between Poincare SUSY breaking and the dS radius. I also review recent work on infra-red divergences in dS/CFT, as well as the phenomenology of CSB. I show that a coincidence between two scales in the phenomenological model is explained by insisting on the existence of galaxies.

KEYWORDS: holography, de Sitter space, supersymmetry.

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1. Introduction

If string theory is to be a theory of the real world, it is crucial to understand how supersymmetry (SUSY) is broken. Much of the work on string theory over the past ten years has ignored this problem, because it is so difficult. More recently, the advent of flux compactifications[1] has allowed various authors to claim a controlled calculation of meta-stable de Sitter (dS) “states of string theory”. The result is that one is inevitably led to a multiple vacuum, eternal inflation picture of the universe, which has been dubbed “the string landscape”. I believe that there are many reasons to be skeptical of these claims and I have documented my skepticism in a series of papers[2].

The purpose of the present paper is not to reiterate these arguments, but to present an alternative theory of SUSY breaking and the cosmological constant, which I have

been pursuing since the beginning of the millenium (actually the fall of 1999). It is currently the only approach to these important problems besides the landscape. Most of what I will present here is not novel, but I will combine various strands of argument that have not appeared together previously. I will also present some new results, which I will describe below.

The method that I have used in analyzing dS space is that of the phenomenologist. I take the robust results of quantum field theory in curved space-time, and treat them as “experimental data” which must be reproduced by a correct quantum theory of gravity in dS space. I begin by outlining the general requirements that a quantum theory of stable dS space must satisfy. In particular, I will review the arguments of Fischler and myself, that the theory has a finite number of physical states. I will then discuss the Hamiltonian, H , of a static observer and see that it must have a number of bizarre properties. I will discuss the extent to which this Hamiltonian can be precisely defined. I will conclude that the eigenspectrum of this Hamiltonian is bounded by the dS temperature. Actual masses of particles and black holes (which are all meta-stable in dS space) are eigenvalues of a Poincare Hamiltonian P_0 which approximately commutes with H for Poincare energy much less than that of the maximal Schwarzschild-dS-Nariai black hole. The requirement that the thermal density matrix for H , coincide with a thermal density matrix for P_0 , for this range of eigenstates, gives a relation between the Poincare eigenvalue and the entropy deficit of the Poincare eigenspace relative to the dS vacuum. This relation is satisfied parametrically by small black holes in dS space. The last result is one of the new features of the present paper¹.

The physics of observational interest in dS space is described by an approximate S-matrix, which is approximately Poincare invariant. In static gauge, the dS generators are not connected simply to the Poincare group. Therefore, I turn to a discussion of global dS space, in an attempt to make contact with quantum field theory in curved space-time. Following [11] I will suggest that the correct quantum formalism provides both a UV and an IR cut-off for the field theory picture. This leads to the idea of quantum deformations of the dS group, first proposed by Rajaraman[21]. The most developed version of this idea is that of Guijosa and Lowe[22], who introduced the critical notion of cyclic representations. I propose a Hilbert space formalism for dS space in which the fundamental variables are fermionic creation and annihilation operators transforming in a reducible representation of the quantum group. I propose a generalization of this to the dS super-group and sketch an algebraic derivation of the scaling law relating SUSY breaking to the cosmological constant, which I conjectured in [6] and argued for in [13]. Some of these algebraic results are new, but incomplete. I

¹This result was known to L. Susskind.

emphasize in this section that generators of the dS group are observer dependent. The generator corresponding to the "static Hamiltonian" in global coordinates, does not have the same spectrum and relation to the Poincare Hamiltonian, as the generator with the same name in static coordinates.

Coming at the problem from the other end, I will describe a program for studying the central claim of cosmological SUSY breaking (CSB) from the point of view of quantum field theory in curved space-time. The initial reaction of most physicists to this claim is that field theory gives no indication of the large renormalization of the classical formula relating the gravitino mass to the c.c. in spontaneously broken SUGRA. I believe that this is because no one has actually studied a gauge invariant definition of the gravitino mass in dS space. I argue that dS/CFT gives us such a definition, to all orders in the usual semi-classical expansion. dS/CFT is incompatible with the claim that dS space has a finite number of states, but this discrepancy is non-perturbative in the c.c. To all orders in perturbation theory, the correlators defined by dS/CFT are invariant under gauge transformations which vanish in the infinite past and future of dS space. The formula relating the scaling property of boundary Green's functions to the bulk mass term, gives us a gauge invariant definition of mass. I argue that the notorious IR behavior of quantum gravity in dS space might give IR divergent contributions to the boundary dimensions. I review preliminary calculations in non-gravitational theories with minimally coupled massless scalar fields, which exhibit such divergent mass renormalizations. If this persists for the gravitino mass in SUGRA, one would have exhibited a large correction to classical formulae. The arguments of [11] then imply that what we have found is an anomalous dependence of the gravitino mass on the cosmological constant. It is not clear that higher order field theory calculations can give the correct Λ dependence.

I end the paper with a review of a recent attempt to find a low energy phenomenological lagrangian which implements CSB[14]. This model involved two new scales in order to fit the experimental bounds on the standard model. The first was the CSB scale, which is determined by Λ , according to $m_{3/2} \sim f_0 \Lambda^{1/4} \equiv \frac{F_G}{m_P}$. The second was the scale M_1 of a strongly coupled gauge theory with gauge group \mathcal{G} , and is supposed to be independent of Λ for small Λ . Both M_1 and $\sqrt{F_G}$ were required to be about $1TeV$. Here I remark that if dark matter is a "baryon" of the \mathcal{G} theory, then these two scales are tied together by requiring the existence of galaxies. The coincidence of scales is the same as the cosmological coincidence of dark matter and dark energy densities, and both are explained by insisting that the theory contain galaxies.

Taken together, these results indicate a clear program for studying the theory of stable dS space more carefully, and suggest that it might lead to phenomenologically attractive, and predictive results.

2. The structure of the static hamiltonian

Quantum field theory in dS space predicts that a time-like observer² can only be in causal contact with some of the states in the QFT Hilbert space. Thus, the local measurements made by this observer can at most infer an impure density matrix. Gibbons and Hawking[26] showed that this density matrix was thermal, with temperature

$$T_{dS} = \frac{1}{2\pi} \sqrt{\frac{\Lambda}{3}} / M_P = (2\pi R)^{-1}, \quad (2.1)$$

where R is the Hubble radius of dS space³. This result is easily understood once we announce that field theory in dS space is defined by the analytic continuation of Euclidean functional integrals on the sphere.

We will have occasion to use different analytic continuations, which lead to different coordinates on Lorentzian dS. The simplest is the static coordinate patch in which we analytically continue the azimuthal angle $\phi \rightarrow i\frac{\tau}{R}$. By a simple spatial change of coordinates, the Lorentzian metric takes on the form

$$ds^2 = -d\tau^2 \left(1 - \frac{r^2}{R^2}\right) + \frac{dr^2}{\left(1 - \frac{r^2}{R^2}\right)} + r^2 d\Omega^2. \quad (2.2)$$

Here $d\Omega^2$ is the metric on the unit two sphere. Since the Euclidean time is periodic with period $2\pi R$, the Lorentzian Green's functions are thermal, with temperature, $T_{dS} = \frac{1}{2\pi R}$.

The other continuation we will use is $\theta_1 = \frac{\pi}{2} + i\frac{t}{R}$, where θ_1 is one of the polar angles on the sphere. This gives global coordinates

$$ds^2 = -dt^2 + R^2 \cosh^2(t/R) d\Omega_3^2, \quad (2.3)$$

where Ω_3 is the coordinate on a unit three sphere.

The Penrose diagram of dS space is the square shown in Figure 1. The global coordinates cover the whole diagram, while the (North) static patch covers the triangle labeled N . This is the static patch associated with an observer sitting at a given point (the North Pole) of the three sphere. There is an equivalent patch for any choice of base point⁴, in particular, the South Pole (the cross hatched patch in the picture). At global

²I use the word observer to denote a large quantum system, which has many observables whose quantum fluctuations can be neglected with a certain accuracy. I will later argue that in dS space this accuracy cannot be made infinite, but for purposes of the present section we can ignore this point.

³We will work mostly in four dimensions. For reasons to be explained below I believe that this may be the only dimension in which a quantum theory of dS space makes sense.

⁴equivalent in the sense of being related by dS isometries, which are, in some sense, just gauge transformations. More on this below.

time $t = 0$, the entire spatial slice is finite, and it is covered by the intersection of this slice with the union of the static patches of North and South poles. Note however that the directions of static coordinate times in the North and South Poles are opposite.

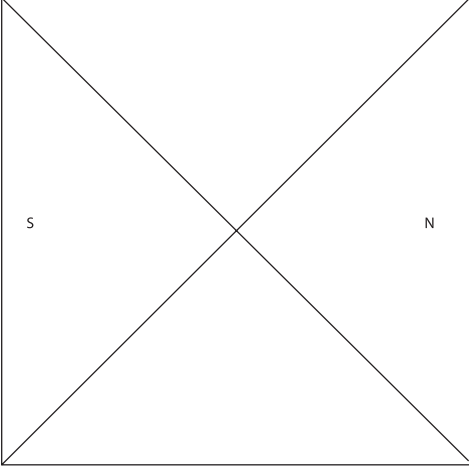


Figure 1: The Penrose Diagram for de Sitter Space: Time Flows Up in the North Causal Diamond and Down in the South Diamond.

This state has $H = 0$. Expectation values of Heisenberg operators acting only on the first tensor factor of the Hilbert space, in this state $|\Psi\rangle$ are equal to time dependent thermal expectation values in the first tensor factor, with density matrix $\rho = e^{-\beta H_1}$.

Manifolds like dS space, and black holes, which have analytic continuations to smooth Euclidean manifolds with compact Euclidean Killing vectors that fix a point, always seem to have analytic extensions in which there is a second copy of the region where the Killing vector is timelike. The second copy has opposite time orientation. For black holes, Israel[9] suggested that quantum field theory on this extended Lorentzian manifold (with generalized Hartle-Hawking boundary conditions) should be interpreted as the thermo-field double of quantum gravity in a static (Schwarzschild-like) patch. This interpretation was extended to AdS black holes by Maldacena[10] and to dS space by Goheer *et. al.* [20]. Note that with this interpretation of the physics, the South pole patch of dS space is no more physical than the extra asymptotically flat universe on the other side of the Einstein-Rosen bridge in the Kruskal-Schwarzschild manifold. The physical Hilbert space of the system is just thought of as the causal diamond of the North Pole.

Another argument which leads to the same conclusion was presented in [6]. In the Euclidean quantization of gravity on manifolds with the topology of a sphere, rotations of the sphere are treated as diffeomorphisms and are gauge fixed. If we want

There is an interesting quantum mechanical interpretation of these geometrical facts, which generalizes an observation about black holes due to Werner Israel[9]. It has recently been extended to AdS black holes by Maldacena[10], and to dS space by Goheer *et. al.*[20]. One can compute thermal expectation values in a quantum system in terms of ordinary quantum expectation values in an extended system called the *thermofield double*. One takes two identical Hamiltonians, H_+ and H_- and, in the tensor product Hilbert space, introduces the Hamiltonian $H = H_+ - H_-$. Now one introduces the state

$$|\Psi\rangle = \sum e^{-\frac{\beta}{2} E_n} |E_n\rangle_1 \otimes |E_n\rangle_2. \quad (2.4)$$

to analytically continue a Killing symmetry of the sphere and use it as the Hamiltonian H , (the causal patch continuation) we gauge fix in such a way that H is left as a symmetry of the quantum Hilbert space. The rest of the generators are treated as diffeomorphisms. We mod out by them and they do not act on the physical Hilbert space (*cf.* light cone gauge quantization of the string). On the other hand, [19] in the standard treatment of ordinary quantum field theory in dS space, by Euclidean methods, we can analytically continue to the causal patch and then obtain Green's functions on the global dS manifold by acting with dS transformations which map the causal patch onto the rest of the manifold. This suggests that we view the region outside of a given causal patch as simply a gauge copy of the region inside of it. This is one origin of the idea of *observer complementarity*[3], to which we will return below.

Now imagine, as everyone does, that in the quantum theory of gravity, there is some kind of UV cutoff of spatial scales. In a theory with finite volume, we would then expect to have a finite number of states. A much stronger argument that the quantum theory of dS space has a finite number of physical states, comes from the observation of Gibbons and Hawking that it appears to be a thermal system, with a finite entropy. We need one extra assumption to prove that the number of states is finite, namely that the energy spectrum of the static Hamiltonian is bounded from above. Here we will give only a naive argument for this, without being very precise about how energy is defined. Below we will argue that it is very likely that the upper bound on the spectrum of the static Hamiltonian is much more stringent than the one we are about to use.

In the QFT approximation, when we look at localized states of very high energy, we inevitably come to a point that the Schwarzschild radius of the state is larger than the radius of the state in the background geometry. At this point the QFT approximation breaks down and states are best described as black holes. The scaling of entropy with size changes. The maximal energy is now completely determined by the size of the state. Most states of a fixed energy, with the minimal size for that energy, are black holes.

In dS space there is a maximal black hole, the Nariai hole, whose cosmological and black hole horizon areas coincide. The metric of the causal patch has the form

$$ds^2 = -\beta^2(r)dt^2 + \frac{dr^2}{\beta^2(r)} + r^2d\Omega^2 \quad (2.5)$$

where $\beta^2(r) = (1 - r^2/R^2)$. Black holes in the causal patch are described by the replacement $\beta^2 \rightarrow \beta_M^2 \equiv (1 - \frac{2M}{r} - \frac{r^2}{R^2})$. β_M has two zeroes, r_{\pm} , for $M > 0$, which locate the positions of the black hole (r_-) and cosmological (r_+) horizons. Semiclassically, both horizons are sources of thermal radiation, and the black hole horizon is hotter, except in the limiting case (the Nariai black hole) in which the two horizons coincide.

This leads to the expectation that black holes decay, which is certainly correct for $M \ll R$. Bousso and Hawking[18] have argued that the same is true for all values of M including the Nariai case. This point is probably worth revisiting, since it will be crucial in the discussion below.

It is also worth noting that the entropy of all of these black hole states is smaller than that of the empty dS vacuum. The total area of the two horizons is smaller than the area of the empty dS horizon, and monotonically decreases as the mass is increased. This again suggests that the black holes should be viewed as low entropy excitations of the dS vacuum, which decay back to it. The behavior of the entropy as a function of black hole mass frustrates an observer's attempt to access all of the states of the system as localized excitations in its horizon volume. As the local entropy is increased, the total entropy gets smaller indicating that an observation of a large amount of localized entropy freezes the rest of dS space into a very low entropy state⁵. The existence of a maximal energy in dS space, coupled with the assumed thermality of the density matrix, and finite entropy, imply that the system has a finite number of states.

Our observations about the decay of black holes, have important implications for the structure of the static Hamiltonian. One might have thought that the black hole mass parameters referred to approximate eigenvalues of the dS Hamiltonian. That is, one imagines an approximate Hamiltonian, H_0 , for which black holes are stable excitations with eigenvalue equal to their mass parameter. Hawking decay would then be the result of a correction to this Hamiltonian, but the diagonal matrix element of the Hamiltonian in the meta-stable black hole state would be approximately equal to the mass. Indeed, Gomberoff and Teitelboim[16] have described a semiclassical formalism in which one can define a conserved generator in dS-black hole space-times, whose value is the black hole mass. I have advocated this as evidence for the above Hamiltonian interpretation in [17]. However, if all black holes decay, this is inconsistent with quantum mechanics⁶. In such a picture, M would be approximately the diagonal matrix element of the true Hamiltonian in the M eigenstate of H_0 . However, the diagonal matrix element of the Hamiltonian is always bounded by its maximal eigenvalue. If all black holes decay, the maximal eigenvalue of H is of order the dS temperature. We conclude that the entire eigenspectrum of H consists of states in the dS vacuum

⁵This conclusion is supported by the extended Penrose diagram of dS black holes. The global geometry has a black hole in both North and South Pole causal patches, consistent with the Israel interpretation as a thermofield double. In the Nariai limit, every observer sees a (generically moving) black hole in its causal patch.

⁶I would like to thank L. Susskind, and particularly M.Srednicki, for arguments, which set me straight on this point.

ensemble⁷.

Classically, these vacuum states have zero energy, but quantum mechanically we might imagine that they have energies as high as the dS temperature. Indeed, a random spectrum of states, spread between 0 and T_{dS} with density $e^{-S_{GH}} = e^{-\pi R^2}$ would provide a heat bath for localized states in dS space, thus explaining why they have a thermal density matrix. The Coleman-DeLucia tunneling amplitudes between two different dS spaces[27], provide further evidence for this identification. Probabilities for the forward and reverse tunneling processes are computed in terms of the same instanton solution, but with a subtraction of the action of the initial dS space for the transition $dS_I \rightarrow dS_F$. The ratio of probabilities for inverse processes is given by the exponential of the difference of the entropies. This is what we expect from the law of detailed balance if the entropy is the dominant term in the free energy. The condition for entropy dominance is that most of the states lie below the temperature.

Similar arguments can be made about the decay of all states of the system which are not black holes but have energies above the dS temperature. There are no arguments that such states are stable. Consider for example an electron in our own universe, assuming that it asymptotes to a dS universe with the nominal value of the cosmological constant indicated by cosmological data. We usually argue that electrons are stable because of charge conservation, but in dS space, the charge of an electron in the static patch is canceled by a tiny charge density uniformly spread over the cosmological horizon. From the point of view of global coordinates, this is a consequence of the fact that spatial sections are compact so the global manifold cannot have a net charge. As a consequence, electrons in a causal patch of dS space are not stable, even if they are bound to the observer, because there is a finite probability for the charge density on the horizon to coalesce as a positron and annihilate the electron. This probability is extremely small, but the electron's lifetime is much shorter than the dS recurrence time, which we will discuss later.

The same argument we used above shows that the electron mass cannot be an approximate eigenvalue of the static dS Hamiltonian. This seems paradoxical, and contradicts the predictions of QFT in a dS background. There are two elements to what I think is the correct resolution of this paradox. The first is fairly conventional. It is well known that the conventional constraint equations of GR in a temporal gauge

$$\mathcal{H} = \mathcal{P}_i = 0, \tag{2.6}$$

can be interpreted as the vanishing of the total energy and momentum densities once gravitational effects are taken into account. Total energy and momentum are pure

⁷It should be noted that if the Nariai black hole were stable, this conclusion would not follow.

surface terms. In the causal patch the only surface on which to imagine defining these objects is a stretched horizon, a time-like surface within a few Planck distances of the horizon. The QFT description of physics breaks down on such surfaces, because of the large blue-shift between the observer and the stretched horizon. Thus, we can easily imagine, in the full theory of quantum gravity, near horizon contributions to the total static energy which almost cancel the QFT contribution. The electron mass we measure might be viewed as an integral over surfaces far enough away from the horizon that these near horizon quantum gravitational corrections are small.

These remarks are a tentative answer to the question of why the electron energy is “really” almost zero, but they do not explain how to define the electron mass we measure in dS space as a mathematical quantity in a holographic formulation of the theory. I think that the answer to this is related to the relation between the approximate Poincare generators, and dS generators of a dS space with very small c.c.

The near (future) horizon geometry of the causal patch is

$$ds^2 = R^2(-dudv + d\Omega^2) \quad (2.7)$$

while that of asymptotically flat space near future null infinity is

$$ds^2 = -\frac{1}{v^2}(dudv + d\Omega^2). \quad (2.8)$$

In both cases the horizon is at $v \rightarrow 0$. The static dS Hamiltonian is the boost $u\partial_u - v\partial_v$ in (u, v) coordinates. Lorentz transformations are conformal transformations of the near null infinity geometry, while the translation generators have the form $P_\mu = Y_\mu(\Omega)\partial_u$, where $Y_0 = 1$ and $Y_i = n_i$, (with $n_i^2 = 1$ parametrizing the sphere⁸).

Now imagine that we have a quantum realization of dS space which, for large R carries both a representation of the static Hamiltonian and an approximate representation of the Poincare algebra. Given our discussion above the static generator should have a spectral cutoff of order (Planck units) $T_{dS} \sim \frac{1}{R}$. The Poincare generator, P_0 will also be bounded, because the system has a finite number of states. Given that the spectrum above the Planck scale is dominated by black holes, we might imagine the cutoff on P_0 is of order R^{d-3} , the maximal black hole mass in dS space. A cartoon version of these generators would be

$$H \sim \frac{1}{R}(u\partial_u - v\partial_v) \quad P_0 \sim R\partial_u, \quad (2.9)$$

with order one cutoffs on the spectrum of the derivative and scaling operators. Then

⁸We have chosen a particular Lorentz frame in which the metric on the sphere at infinity is round.

$$[H, P_0] \sim \frac{1}{R} P_0 \quad (2.10)$$

Thus, for large R , the commutator is small for states whose P_0 distribution contains only eigenstates with energies $\ll R$. For states satisfying this constraint, P_0 becomes an additional approximate quantum number, whose spectrum breaks the huge degeneracy of H . This equation could also explain why P_0 eigenstates are approximately stable under the time evolution defined by H . Masses of meta-stable particle and black hole excitations of the system would be related to P_0 eigenvalues.

Note that it is only in four dimensions that the P_0 generator approximately commutes with the static generator for energies all the way up to the maximum black hole mass. For $d > 4$ the generators fail to commute for parametrically smaller values of P_0 . The stability of large black holes could not be explained by approximate conservation of P_0 . This might be an indication that a quantum theory of dS space may only make sense in four dimensions, or that the commutation relation is modified in higher dimensions.

This picture implies an interesting relation for the expansion coefficients of P_0 eigenstates in the basis with H diagonal:

$$\sum |p_0 \rangle \langle p_0| = \sum P_n(p_0) |h_n \rangle \langle h_n| : \quad H|h_n \rangle = h_n |h_n \rangle. \quad (2.11)$$

The sum on the left hand side of this equation could be either the degeneracy of the eigenvalue p_0 , or the density matrix of states within an interval of size T_{dS} , centered on p_0 . The density matrix is $e^{-\frac{H}{T_{dS}}}$. QFT tells us that for eigenstates of P_0 for which a QFT description is valid (including at least black holes whose radii grow more slowly than R as $R \rightarrow \infty$) the probability of finding $|p_0 \rangle$ is approximately $e^{-\frac{p_0}{T_{dS}}}$. Thus

$$\frac{\sum P_n(p_0) e^{-\frac{h_n}{T_{dS}}}}{\sum e^{-\frac{h_n}{T_{dS}}}} = e^{-\frac{p_0}{T_{dS}}}. \quad (2.12)$$

An easy way to get this result is to write the Hilbert space as a tensor product. Thus, we break the index n up into a pair of indices $n = (k, L)$. Assume that $\sum_k = e^{\frac{p_0}{T_{dS}}}$ and that $P_{k,L} = \delta_{k1}$. We will also approximate $e^{-\frac{h_n}{T_{dS}}} \approx 1$. Thus we get a relation between the p_0 eigenvalue and the entropy deficit of this eigen-space relative to the whole Hilbert space.

An exactly similar relation appears for black hole entropy for masses $\ll R^{d-3}$. The cosmological horizon for such black holes occurs in static coordinates at

$$r_+ \approx R - \frac{8\pi M}{(d-2)A_{d-2}R^{d-4}}. \quad (2.13)$$

The area deficit, relative to empty dS space is

$$\Delta A = (d-2)A_{d-2}MR \quad (2.14)$$

where A_{d-2} is the area of the unit $d-2$ sphere. The relation between entropy and mass given by this equation, coincides with the one derived quantum mechanically in the previous paragraph[4]. Interestingly, the same argument works for charged black holes. The entropy formula depends on the charge, but the leading correction for $R_S \ll R$ does not. Note that this is a requirement for agreement with the thermal formula from quantum mechanics, which makes no reference to the charge.

In [17] I reported on a formulation of dS quantum mechanics in terms of fermion operators (see the next section) and proposed to use this definition of mass in terms of entropy to define the static Hamiltonian. This is now seen to be wrong, but it may be possible to consider that construction to be a model of the black hole eigenstates of the Poincare Hamiltonian .

The basic idea of that model, which was constructed in collaboration with B. Fiol, was simple: introduce fermion operators

$$[\psi_i^A, (\psi^\dagger)_B^j]_+ = \delta_i^j \delta_B^A$$

which are $N \times N + 1$ matrices. These are “operator valued sections of the spinor bundle” over the holographic screen on the cosmological horizon of a given observer[31] and represent quantized pixels on the screen. As we will see in the next section, N is proportional to the dS radius.

We work in the approximation in which the static Hamiltonian is the unit matrix, and will not reproduce the commutator of static and Poincare generators or the instabilities of black holes. The idea is to implement the principle of *asymptotic darkness*: start from an approximation to quantum gravity in which the simple states are stable black holes, which dominate the high energy spectrum, and introduce black hole instabilities as a perturbation. We want to describe the basis of black hole eigenstates of the Poincare Hamiltonian.

The entire Hilbert space is the dS vacuum ensemble of the static Hamiltonian. We identify black hole states by choosing an integer $N_- < [\frac{N}{2}]$. Now, in some specific basis for the fermionic matrices, make a block decomposition

$$\begin{pmatrix} \Psi_+ & \Psi_{+-} \\ \Psi_{1+} & \Psi_{1-} \\ \Psi_{-+} & \Psi_- \end{pmatrix}.$$

Ψ_\pm is an $N_\pm \times (N_\pm + 1)$ matrix, with $N_+ = N - N_- - 1$. $\Psi_{1\pm}$ is a $1 \times N_\pm$ matrix, and Ψ_{-+} and Ψ_{+-} are $N_- \times N_+ + 1$ and $N_+ \times N_- + 1$ matrices respectively. Black hole

states are identified by the constraint

$$(\Psi_{-+})_i^A |BH\rangle = \Psi_{1-} |BH\rangle = 0. \quad (2.15)$$

The states created by Ψ_+ and Ψ_{+-} creation operators should be identified with the cosmological horizon in the presence of the black hole, while those created by $(\Psi_-)^\dagger$ operators are associated with the black hole horizon. The states created by $(\Psi_{-+})^\dagger$ operators should perhaps be associated with particle states propagating in the space between the two horizons, but we will see that this does not account for the entropy of such states. The $1 \times N_\pm$ matrix fermions do not have any obvious macroscopic interpretation.

The equations for the horizons of a Schwarzschild dS black hole may be put in the form

$$\begin{aligned} R^2 &= (R_+ + R_-)^2 - R_+ R_- \approx R_+^2 + R_+ R_- \\ 2MR^2 &= R_+ R_- (R_+ + R_-) \approx R^2 R_-, \end{aligned}$$

where the approximate forms are good in the limit $R_+ \gg R_-$. The entropy deficit of a small black hole is $\pi R R_- = 2\pi R M$. In the fermion model the entropy deficit is approximately NN_- for $N \gg N_-$, so if we identify (in Planck units) $\sqrt{\pi}R = N$ and $\sqrt{\pi}R_\pm = N_\pm$, our model reproduces the “data”. We also obtain the identification $M = \frac{1}{2\sqrt{\pi}}N_-$.

In order to write an operator form for the Poincare Hamiltonian, we must choose a matrix basis for the maximal size black hole $N_- = \lfloor \frac{N}{2} \rfloor$. Black holes of a given size are constructed by imposing the above constraint for a given value of N_- less than the maximum. The Poincare Hamiltonian is given by

$$M = \frac{1}{2\sqrt{\pi}} \left(\lfloor \frac{N}{2} \rfloor - \sum_{j,A} [(\Psi_{-+})^\dagger]_A^j [\Psi_{-+}]_j^A \right),$$

where $1 \leq j \leq \lfloor \frac{N}{2} \rfloor$ and $1 \leq A \leq N - \lfloor \frac{N}{2} \rfloor - 1$. Note that this formula does correctly reproduce the fact that what we have called particle excitations of a small black hole make positive contributions to the energy, but it does not reproduce their spectrum or entropy. I view this as crudely analogous to the way that the free quark gluon Hamiltonian correctly describes generic high energy excitations of QCD, but fails to reproduce the hadron spectrum.

An important feature of the above construction is that our description of the black hole spectrum involved a choice of basis for the fermionic matrices. Other choices will give the same results, but the states that one “observer” associates with a localized black hole, will be mixed up with the cosmological horizon states of an “observer” who

makes a different choice of basis. It is very tempting to identify this ambiguity as the quantum analog of the choice of static coordinate frame in classical de Sitter space.

To summarize, it appears that the spectrum of the static dS Hamiltonian is cut off at an energy of order $1/R$, with a density of states of order e^{-S} where S is the dS entropy. e^S is not exactly the number of states, because the density matrix is thermal. However, since most of the states are at energies below T_{dS} , the discrepancy between e^S and the number of states is not large, and goes to zero as $R \rightarrow \infty$. The huge degeneracy of dS horizon states is broken by another operator, P_0 , whose commutator with the static Hamiltonian is small in the subspace spanned by eigenvectors of P_0 with eigenvalue $\ll R$. P_0 is one of the generators of a super-Poincare algebra, which emerges in the limit $R \rightarrow \infty$. The requirement that the thermal density matrix of the static Hamiltonian also gives thermal statistics for the Poincare generator, implies a relation between the Poincare eigenvalue p_0 , and the entropy deficit relative to the dS vacuum of the p_0 eigenspace. This relation coincides with that derived from the Bekenstein-Gibbons-Hawking formula for small dS black holes. We constructed an explicit quantum model, which incorporated this relation.

An important consequence of this discussion, is that the information, which is of concern to particle physicists, is encoded in the Poincare Hamiltonian, rather than the static dS Hamiltonian. The two are related by the commutation relation $[H, P^0] \sim \frac{1}{R}P^0$. As we will see, the part of the P^0 spectrum where P^0 is an approximately conserved quantum number under H evolution, accounts for only a tiny fraction of the states in the Hilbert space. The rest of the states are not localizable in the observer's horizon, and form a degenerate soup which is localized close to the horizon. From the static observer's point of view⁹ the description of the rest of the Hilbert space is somewhat arbitrary. It must satisfy some weak constraints, which ensure that the states on the horizon thermalize the localizable degrees of freedom at the right temperature. In later sections we will see how to obtain a more constrained description of dS space by choosing a basis for the Hilbert space which simultaneously describes localizable states in disjoint, causally disconnected horizon volumes.

2.1 Measurement theory in dS space

Working physicists usually try to avoid thinking about the arcana of quantum measurement theory. Discussions of these issues often smack of academic philosophy, and we all know what we really do to measure something anyway, right?

I'm afraid that in the quantum theory of gravity we really have to address these issues. What is more, discussion of them illuminates certain otherwise obscure features

⁹Which means from the point of view of any realistic measurements.

of the theories that we know to make mathematical sense, namely the fact that the only gauge invariant observables are boundary correlators. Finally, I will argue below that a question which properly belongs to the philosophy of science has some relevance for decisions about whether a quantum theory of dS space can ever make sense.

Anyone who has ever thought about quantizing generally covariant field theories knows that there is a problem of interpreting the observables in terms of local physics on the world volume of the universe. In $0+1$ and $1+1$ dimensions, where Wheeler-DeWitt quantization makes sense beyond the semiclassical approximation, the observables refer to global properties of multiple disconnected “universes” (world lines or world sheets). They do have an interpretation as the perturbation expansion of a Scattering matrix (and in the particle case the local correlation functions) of an external space-time in which the particles and strings are embedded, but no interpretation in terms of approximately local physics on the world volume.

Hawking suggested many years ago[28] that the S-matrix would be the only gauge invariant observable in a theory of quantum gravity in asymptotically flat space-time¹⁰. Modern developments, particularly the Fischler Susskind Bousso covariant entropy bound suggest a deep reason for this, which is connected to quantum measurement theory.

Quantum measurement theory can be summarized in a few lines in the following way: Certain large quantum systems, in particular cut-off quantum field theories, have *pointer observables* whose quantum fluctuations can be made very small. The canonical example is the volume averaged value of a local field. There are states of the system in which the quantum fluctuations of such pointer observables are arbitrarily small in the limit of infinite volume. The system has a large number of states with essentially the same value for the pointer observable. Typical tunneling amplitudes from one value of such a *pointer observable* to another, are of order e^{-VM^3} where M is the energy cutoff and V the volume over which the observable is averaged.

Quantum mechanics makes precise mathematical predictions for any system. The operational meaning of these precise predictions is extracted by coupling the original system to a measuring apparatus in such a way that the value of a microscopic observable A is correlated with the value of a pointer observable of the apparatus. These measurements are robust over time scales of order the inverse pointer tunneling probability per unit time¹¹. Infinite precision and robustness are attained by taking the limit of an infinite measuring apparatus.

¹⁰And then proceeded to argue that the S-matrix would be replaced by the Dollar Matrix.

¹¹as long as we do not make another measurement on the same system, which measures a variable B that does not commute with A .

These ideas are problematic in a theory of gravity, because infinite machines have infinite gravitational interaction with the measured system. The only way to resolve this problem is to make the measurements at infinity, and this is the reason that string theory, the only quantum theory of gravity which specifies a complete set of gauge invariant observables, only makes predictions about scattering amplitudes and other kinds of boundary correlators, in space-times which are globally foliated by infinite volume spatial sections.

In a theory of a stable dS universe, we do not have the luxury of an infinite boundary of space on which to make measurements. Indeed, I have argued here that we can describe an entire stable dS universe with a finite number of physical states. *This means that the theory of such a space-time has an inherent quantum uncertainty built in to it.* The theory cannot self consistently describe measurements of its predictions with a greater accuracy and robustness than some fixed finite bound. This means, that there cannot be a unique mathematical description of the theory. Two mathematical models, whose predictions differ by an amount smaller than this *a priori* bound on the precision of measurements, will not be operationally distinguishable. Predictions over time scales greater than the tunneling time for the most robust pointer observable that can be manufactured from the ingredients at hand, are meaningless.

There is a subtle point of scientific philosophy inherent in the statements of the previous paragraph. It has to do with the precise relation between mathematics and the physical world. The mathematical elegance of the known laws of physics, has lead many researchers to the (at least subconscious) conclusion that the relation is Platonic/Pythagorean. That is, mathematics has some kind of existence outside the world of measurement, so that the predictions of a mathematical theory should be discussed and taken seriously, even when we cannot, in principle, devise a method for testing them.

My own point of view is quite different. Mathematics is a creation of human beings, who are physical objects in a world we can know about only through measurement. We can only discuss those of its predictions which are, at least in principal, subject to verification by physical observation. Mathematics produces models. If they are models of the entire universe then they must supply us with a self consistent set of instructions for testing the model. Predictions which go beyond the model's ability to "self-test" have no physical meaning.

In [11] we proposed that this should be viewed as a kind of gauge invariance: two Hamiltonian descriptions of *e.g.* the static coordinate patch of dS space are equivalent if their predictions for all observables agree within the intrinsic limitations on the

accuracy of measurements¹².

With the philosophical baggage out of the way, we can try to discuss what the quantitative bounds on precision in dS space might be. There are three qualitatively different types of states in the dS Hilbert space, when it is viewed from the perspective of the static patch observer. The first class of states are well described by quantum field theory in the static patch. To get a qualitative idea of their number one notes that most of the states in a 4 dimensional QFT are described by the conformal fixed point theory. We must cut-off the field theory at some scale M and put it in a box of radius $\sim R$, so the entropy of field theoretic states is of order $M^3 R^3$. The energy of a typical state of this type, will be of order $M^4 R^3$, and this is also the Schwarzschild radius of the state in Planck units. Insisting that $R > R_{Schw}$ we find $M < R^{-\frac{1}{2}}$, which implies that the entropy in field theoretic states is $< R^{3/2}$ in Planck units.

The second class of states are the states on the cosmological horizon. In fact, as we have discussed above, these are the only absolutely stable eigenstates of the static patch Hamiltonian. Their entropy is of order R^2 . These states are not useful for making a measuring apparatus for the static observer. A static observer can only access them by probing a region of his coordinate patch with very large quantum/thermal fluctuations.

Finally, we have horizon states of black holes localized within a causal patch. Their entropy is also of order R^2 (for black holes of order the horizon size), though with a coefficient one third of the dS vacuum entropy. The No-Hair theorem suggests that it is not possible to build robust pointer observables from black hole eigenstates. Although they are numerous, they are much more degenerate than states of a quantum field theory, and the locality arguments, which guarantee small tunneling amplitudes between different macrostates of a QFT, do not apply to them. Of course, our understanding of black hole microstates is still too primitive to make definite conclusions about this point.

Assuming that robust measuring devices can be constructed only from QFT states, we can draw two important conclusions about the description of physics in the static patch of dS space. The first is that we cannot access most of the states of the theory by measurements. QFT can only tell us inclusive things about the horizon states. The semi-classical analysis that we have reviewed above suggests that, as far as QFT states are concerned, the rest of dS space imitates a thermal bath at the dS temperature. Thus, models of the static Hamiltonian which differ only in their description of the horizon states will be essentially equivalent if they couple the horizon states to the QFT states in a way that is compatible with thermalization. The second point is that the

¹²It is becoming more and more tempting to imagine that this *quantum measurement gauge equivalence* lies at the root of the general covariance of classical GR. That is, we will eventually derive the latter principle from the former.

tunneling time for the most robust QFT devices is, for large R , an infinitesimal fraction of the dS recurrence time[29]. Indeed, for the value of R indicated by cosmological observations in the real world, the recurrence time is essentially the same number in Planck units ($e^{10^{123}}$) that it is in units of this tunneling time ($e^{10^{123}-10^{92}}$). These two remarks are connected. The recurrence time is related to the splitting between the levels of the horizon states. Many different Hamiltonians for the horizon states will give the same results for physics measured by QFT devices, but predict different horizon configurations over a recurrence time. It is only by paying attention to these unobservable properties of the model that we can attribute reality to the recurrence time scale.

The considerations of this section lead to another interesting observation. The total number of degrees of freedom, which can be described by local field theory in a fixed horizon volume, is much less than the total entropy of dS space. This means that the dS Hilbert space contains enough states to describe of order $R^{1/2}$ horizon volumes with independent (commuting) degrees of freedom. In the global picture of dS space in QFT, the system at late global time seems to have an infinite number of copies of the degrees of freedom of a single horizon. These considerations suggest an IR cutoff on the QFT Hilbert space. Crudely speaking, this is an upper cutoff on the value of global time, of the form $(\cosh(t_{max}/R))^3 \sim (RM_P)^{1/2}$. Of course, if we are willing to reduce the UV cutoff on our field theoretic states, we can use a QFT description of most of the states at larger values of the global time¹³.

In the next section, I will try to give a more precise definition of a finite system which might converge, as $R \rightarrow \infty$, to the QFT picture of dS space in global coordinates, at a global time where there are of order $(RM_P)^{1/2}$ horizon volumes in the spatial slice.

3. Global coordinates and local physics

We have seen that a local observer in dS space can only access a limited fragment of the information content of the space-time. This implies that there is large equivalence class of choices for the Hamiltonian of the static observer, which will give rise to the same predictions for experiments we are likely to do. It is reasonable to search for members of this equivalence class, which are simple and mathematically elegant, and make choices about how to describe those aspects of the physics a local observer will not access.

¹³We can also use QFT with a relatively large cutoff, but insist that the system is in its vacuum over most of space-time. All of these descriptions should eventually be thought of as different gauge choices.

The ambiguity in the quantum description of dS space is related to the question of what the $SO(1, 4)$ isometry group of dS space means. Witten[7] has argued that it is just a group of gauge transformations, which does not act on gauge invariant physical states. Our arguments suggest that there are no precise measurements in dS space, and that the most precise measurements one can contemplate are tied to the frame of a particular local observer. Thus, it seems clear that the $R \times SO(3)$ generators that preserve a given causal patch, are to be viewed as global symmetry generators. We have seen however that much of the interesting local physics is encoded into the Poincare Hamiltonian, which is only an approximate symmetry, rather than the static Hamiltonian. A more global description of dS space leads to the possibility of a more elegant description of the physics, in which the Poincare Hamiltonian is more closely tied to the dS group.

In a global description of dS physics, the arguments of [11] and of the previous section, suggest that there will be something resembling a field theoretic description of all states. In field theory dS isometries converge to the Poincare group, and the distinction between the generators that we introduced in the static gauge is no longer apparent. Recall that the description of the relation between static and Poincare Hamiltonians was motivated by the behavior of these generators on the horizon. A global description should have no trace of a given observer's cosmological horizon. It depicts most of the states of the system as (cutoff) localizable field theory states on the dS manifold at a time sufficiently later than $t = 0$, that they do not form black holes. The two descriptions can only agree (approximately - but within the intrinsic limit of measurements in dS space we can't tell the difference) for local physics within a single horizon.

So we will try to match, as closely as possible, the formulation of quantum field theory in the global coordinate system on dS space. QFT, even with some sort of UV cutoff, appears to describe an infinite number of degrees of freedom in dS space. Any correct formalism will provide some kind of infrared cutoff, and produce a finite number of states.

We begin by recalling the thermofield double interpretation of the Euclidean vacuum state in dS space. If we consider the generator of the dS group H corresponding to a fixed boost in $SO(1, 4)$ ¹⁴, the thermofield interpretation suggests that we write $H = H_+ - H_-$, where H_{\pm} are positive definite and represent the Hamiltonians of the North and South causal diamonds.

However, this does not mean that the generators H_{\pm} must coincide with the static Hamiltonian of the previous section. In fact, we will see that they are more closely

¹⁴Readers who are worrying about the fact that $SO(1, 4)$ has no finite dimensional unitary representations, are urged to hold on until the next subsection.

related to the Poincare generators of the local observer.

In particular, since a global coordinate description should make no reference to the cosmological horizon, our analysis of the commutation relations between the static and Poincare generators is no longer applicable. In global coordinates we should instead expect that the two groups are related by contraction in almost the usual way. The equivocation “almost” in the previous sentence refers to the fact that we expect Poincare energies to be bounded from below. Thus the correct relation is

$$H_{\pm} \rightarrow RP_{\pm}^0.$$

This then allows us to imagine SUSY generators satisfying

$$[Q_{\alpha}^{\pm}, \bar{Q}_{\dot{\beta}}^{\pm}]_{+} = \sigma_{\alpha\dot{\beta}}^{\mu} P_{\mu}^{\pm},$$

for $R \rightarrow \infty$.

We do not expect these generators to commute with the Poincare generators for finite R . Instead the commutator should go to zero only in the $R \rightarrow \infty$ limit. In order to get some insight into the form of the non-zero commutator, we consider the super-dS group. The four dimensional Majorana representation of the Dirac matrices has the following properties:

$$\begin{aligned} [\gamma^{\mu}, \gamma^{\nu}]_{+} &= \eta^{\mu\nu} \\ (\gamma^{\mu})^{*} &= -\gamma^{\mu} \\ (\gamma^{\mu})^T &= -\gamma^0 \gamma^{\mu} \gamma^0. \end{aligned}$$

That is, all matrices are imaginary, γ^0 is antisymmetric, and the others are symmetric. η is the “mostly minus” four dimensional Minkowski metric. $\gamma^4 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ is another imaginary anti-symmetric matrix which completes the algebra of $\gamma^M = (\gamma^{\mu}, \gamma^4)$ to

$$[\gamma^M, \gamma^N]_{+} = \eta^{MN},$$

the $SO(1, 4)$ invariant Clifford-Dirac algebra. The matrices $\gamma^0\gamma^{\mu}$ and $\gamma^0\gamma^4\gamma^{MN}$ (where doubly indexed γ matrices are anti-symmetrized products of singly indexed ones), are all symmetric. The super $SO(1, 4)$ algebra is

$$[q_{\alpha}, q_{\beta}]_{+} = (\gamma^0\gamma^4\gamma^{MN})_{\alpha\beta} J_{MN},$$

where J_{MN} are the $SO(1, 4)$ generators. Although group theory allows the q_{α} to be real, there is a well known problem with representing this algebra with Hermitian q_{α} , since there are no highest weight generators of the dS algebra. Instead, we will write

$$q_{\alpha} = \sqrt{R}(Q_{\alpha}^{+} + T^{\alpha\beta}Q_{\beta}^{-}),$$

where T represents a time reversal operation and satisfies $TT^T = -1$. The operators Q_α^\pm are $SO(1, 3)$ Majorana spinors and satisfy the algebra

$$[Q_\alpha^+, Q_\beta^-]_+ = 0$$

$$[Q_\alpha^\pm, Q_\beta^\pm]_+ = (\gamma^0 \gamma^\mu)_{\alpha\beta} P_\mu^\pm$$

The anticommutation relations of the large and small q supercharges are consistent in the large R limit if we take $J_{0\nu} = R(P_\nu^+ - P_\nu^-)$, and assume that the $SO(1, 3)$ generators are bounded by something of order the P_μ generators.

Note that the super dS algebra (actually its q deformation) is represented only in the full thermofield double Hilbert space. It is amusing that the thermo-field double interpretation of dS space provides us with a resolution of the old problems of realizing the dS supergroup in quantum theory. In order to obtain a symmetry group with finite dimensional unitary representations, I propose to q -deform the dS super-algebra. We will discuss q -deformation in the next section, but we note that the operation of q -deformation does not change the commutators of Cartan generators with raising and lowering operators. We will use these undeformed commutators to estimate the scale of SUSY breaking.

In particular each component of the SUSY generators Q would have fixed weights, w , under commutation with $H^+ - H^-$.

Let us combine this algebra with our expectations for the spectrum of P_0 based on the previous section. There we argued that the global QFT picture of dS space might be valid over a region encompassing of order $R^{1/2}$ horizon volumes as long as we restricted attention to states well described by QFT. Those states had entropy of order $R^{3/2}$ per horizon volume and energy per horizon volume of order R . The total energy is extensive, and is of order $R^{3/2}$. Thus, our Poincare generators P_0^\pm should be finite dimensional matrices whose operator norm is of order $R^{3/2}$. The operator norms of Q^\pm are thus of order $R^{3/4}$. From the fact that the weight of components of the SUSY charges is R independent, and the relation between the dS generators H (in global gauge) and the Poincare generator P^0 , it follows that

$$[P_0^\pm, Q^\pm] = \frac{w}{R} Q^\pm \tag{3.1}$$

Consider a pair of normalized boson and fermion “particle” states, localized in a given horizon volume. These are actually members of a large ensemble of equivalent states. In static gauge this is the vacuum ensemble, with one meta-stable excitation. In global gauge, we describe it as states consisting of one excitation in a single horizon volume plus generic field theoretic excitations in of order $R^{1/2}$ disjoint horizon volumes.

The typical P^0 eigenvalue in this ensemble is of order $R^{3/2}$. The masses of single particle excitations must be extracted by an approximate decomposition: $P^0 \approx \sum P_A^0$, where the sum is over the $N^{1/2}$ horizon volumes. Similarly $Q \approx \sum Q_A$. It is the individual Q_A operators which transform a state with one particle in horizon volume A into the state with one superpartner in the same horizon volume, without affecting the states in the disjoint volumes. Write the commutator

$$[P_0, Q_A] = \frac{w}{R} h_A^B Q_B. \quad (3.2)$$

Consistency with the dS supergroup relation implies $\sum_A h_A^B = 1$. Locality would suggest $h_A^B \sim \delta_A^B$, but there is no reason for locality to apply to this correction to the $R \rightarrow \infty$ limit. Instead, I postulate that the matrix elements of h along a fixed row are all of order 1, with the same phase. $\sum_A h_A^B = 1$ is achieved by cancelations within each column of h . Thus $[P_0, Q_A] \approx \frac{w}{R} Q$. The matrix elements of Q in a typical state of the ensemble are of order $R^{3/4}$, from which we conclude that the typical splitting between superpartner masses is of order $R^{-1/4}$. This is precisely the formula $\Delta m_{SUSY} \sim \Lambda^{1/8} M_P^{1/2}$, for the maximal scale of SUSY breaking in dS space, which I conjectured in [6].

The gravitino will be special, because the zero momentum gravitino state will be related to the action of Q^+ on the vacuum. I have not been able to derive the scaling law for the gravitino mass from this information alone. Of course, if the model is compatible with low energy field theory in a single horizon volume (which we have assumed but not demonstrated in the previous discussion), the scaling law for the gravitino follows from that for the maximal SUSY splitting by the usual Ward identity. In general, one might expect a class of particles whose masses scaled like that of the gravitino.

It is worth devoting a few more words to the violation of locality implicit in our ansatz for h_A^B . Locality in the sense of commutation of operators at spacelike distances, applies to systems that can be described by quantum field theory at all times. This is not true of our global presentation of dS space. It is important to realize that the time evolution operator in global time is *not* a member of the dS group. Furthermore, our discussion of measurement theory makes clear that at global time $t = 0$, most of the states of the system cannot be described by field theory. The entropy in localizable states is only about $R^{3/2}$. The rest of the states interact with the localizable ones with about equal weight. At a much later time we can describe all of the e^{R^2} states by quantum field theory, but we have no more reason to assume that the commutator of the global P^0 with the local Q_A depends only on nearby horizon volumes. The Q_A are not (simultaneously) defined at $t = 0$, and are evolved from more complicated operators at that time symmetric point. At $t = 0$ one can define Q_A for one particular horizon volume. The rest of the states are thermal excitations on that horizon. The global Poincare generator is not a particularly transparent operator at that time.

It is interesting to note that one can, at the level of precision about the algebraic details that was used above, generalize this discussion to arbitrary dimension. If one uses four dimensional formulae to relate the gravitino mass to the maximal SUSY splitting, one obtains a prediction which does *not* agree with the result of [13]. Of course, there is a glaring lacuna in this argument. Low energy, positive metric SUGRA in dimension higher than 4 does not admit solutions with spontaneously broken SUSY and de Sitter vacua¹⁵. Thus, there is no consistent low energy way to extract a prediction for the gravitino mass from that of SUSY matter multiplets in $d \geq 5$ with positive c.c. (which is why we had to use a four dimensional relation above). As I emphasized in [6][14], I view this as an argument that dS space is a strictly four dimensional phenomenon¹⁶.

In summary, the thermofield double interpretation of dS space, enables us to see how a representation of the dS supergroup could be compatible with the positivity of energy. We also saw that this algebraic structure could lead naturally to the CSB scaling law for SUSY breaking. The crucial point here was the assumption that the commutator between the Poincare Hamiltonian and the restriction of the SUSY generators to one horizon, was proportional to the full SUSY generator. This is equivalent to saying that the breaking of SUSY can be attributed primarily to states outside (or on) the cosmological horizon of a given observer.

3.1 Quantum groups

It is a trivial mathematical observation that a system with a finite number of states cannot carry a unitary representation of the non-compact dS group. In the fall of 1999 A. Rajaraman suggested to me that a q-deformed version of the dS group might solve this problem. B. Zumino confirmed that his student, H. Steinacker[30], had indeed shown that q-deformed $SO(2, d)$ groups had finite dimensional unitary representations, when q is a root of unity. Steinacker's work depended in a crucial way on highest weight

¹⁵This statement is much stronger than the usual no-go theorems which have been made by string theorists. We do not restrict attention to highly supersymmetric low energy Lagrangians, but to the most general, minimally SUSic Lagrangian in $d \geq 5$, with positive metric excitations. We do exclude solutions of the form $dS_d \times N_p$, with N non-compact. If there are quantum theories of such space-times, they have an infinite number of states.

¹⁶The question of 2 and 3 dimensional dS spacetimes is more confusing. There are low energy SUGRA lagrangians with dS solutions. However, if dS quantum theories have a finite number of states, they only make unambiguous predictions in an asymptotic expansion in small c.c. The predictions with minimal ambiguity are those for quantities which approach scattering matrix elements at center of mass energy fixed as the c.c. goes to zero. It is not clear that there are any super-Poincare invariant gravitational scattering matrices in less than four dimensions, so the small c.c. limit is much harder to understand.

representations, and it was not clear to me how to generalize it to the dS group. This problem has recently been solved by Guijosa and Lowe[22]. The crucial observation is that, since only periodic functions of the Cartan generators appear in the q deformed algebras, we can have finite dimensional representations which are not highest weight. These quantum group representations are called *cyclic*. Guijosa and Lowe showed that in $1+1$ and $2+1$ dimensional dS space, there is a sequence of cyclic representations of the q deformed dS algebra, which converges, as $q \rightarrow 1$ to each principal series unitary representation of the dS group. The principal series representations are precisely those which appear in QFT in dS space.

In order to utilize these ideas to build a quantum model of dS space, I will use a construction of quantum groups in terms of creation and annihilation operators, due to Polychronakos[5]. This construction uses a different definition of the co-product than that widely used in the mathematics literature, but it appears to be completely consistent when the quantum group is $U_q(N)$ and the creation operators transform in the fundamental. I will want to use it for $SO_q(1,4)$ with creation operators transforming in a reducible cyclic finite dimensional unitary representation of this group. q will be an N th root of unity.

We will determine N in terms of R by noting that $SO_q(1,4)$ has an $SO_q(3)$ subgroup which we should think of as rotation of the dS horizon into itself. The dS horizon is a holographic screen and we should think of the finiteness of the dS Hilbert space as arising from the pixelization of this screen at the Planck scale. In[31] I argued that quantum pixels of a holographic screen were fermionic operators, which transformed in a spinor representation of the transverse rotation group. In spherically symmetric situations, they should form a finite dimensional approximation to the spinor bundle over the sphere¹⁷If there are k fermion operators there will be 2^k states, so k should scale like the area of the sphere. To obtain a construction which realizes this in four space-time dimensions label the fermion annihilation operators as $N \times N + 1$ matrices, Ψ_i^A , with $N \sim R$ in Planck units. The $SO(3)$ group acts on this in the tensor product of the $[N]$ and $[N + 1]$ dimensional representations. This is the direct sum of all half integer spin representations up to spin $N + \frac{3}{2}$. There is an irreducible action of $SU_q(2)$ in each of these spin spaces, with $2\pi(\ln q)^{-1} = N + 1$.

A question which I have not resolved is whether or not there is, for each N , a unique and natural unitary representation of $SO_q(1,4)$ on the space of fermion indices (i, A) , which preserves the anti-commutation relations

$$[\Psi_i^A, (\Psi^\dagger)_B^j]_+ = \delta_i^j \delta_B^A$$

¹⁷That is, each fermion operator should be thought of as being assigned to a given quantized area on the sphere, and transform as a spinor under the local tangent space rotations.

I will assume that there is, and call this representation \mathcal{R} . Presumably, a way to search for it is to find an appropriate sub-algebra of the $SL_q(N(N+1))$ which acts on the fermion labels. We can then use Polychronakos'[5] construction to extend the action of $SO_q(1,4)$ to the entire fermion Fock space.

Now note that since the fermion creation and annihilation operators transform in the representation \mathcal{R} , we can write the Fock space as a tensor product of states with positive and negative weights of $H \equiv J_{04}$. I will interpret this as the splitting of the Hilbert space into states of the northern and southern causal diamonds in dS space. In (by convention) the northern diamond, H will have only positive weights.

We now come to a question which is both extremely interesting and completely confusing. In the limit in which N (and therefore R_{dS}) go to infinity, does the generator H of the quantum group approach the static H of the previous section, or P_0 the Poincare generator? On the one hand, we have argued that in the static patch gauge, the distinction between these two generators is connected to physics very close to the horizon of a given observer. In QFT the global gauge physics does not single out any horizon. In particular, a global observer should, in some approximation, agree with the QFT prediction that there is nothing special, like a buildup of the density of localized states, in the region of the horizon of any given time-like geodesic.

On the other hand, the q-deformation of the group *has* picked out a favored member of the conjugacy class of H by choosing a particular split into Cartan generators and raising and lowering operators. Note that, as far as H is concerned, a similar split is chosen by the contraction of the dS group to the Poincare group (We choose $P_0 = \frac{1}{R}J_{04}$ for some particular non-compact Cartan generator in $SO(1,4)$).

I will take the point of view that the operator H , periodic functions of which appear in the quantum dS group, converges to RP_0 in the $N \rightarrow \infty$ limit. Similarly the q-deformed J_{i0} generators should converge to RP_i , where P_i are the Poincare momentum generators. Thus, with the association between N and R discussed above, the $q \rightarrow 1$ limit of the quantum algebra is not the dS algebra $SO(1,4)$, but its contraction, the Poincare algebra $ISO(1,3)$.

More precisely, we envisage the above limiting procedure taken only for the generators H_{\pm} , where $H = H_+ - H_-$ and the individual operators are the positive generators in the thermofield double interpretation of the Hilbert space. That is, H_{\pm} acts only on the tensor factor of the Hilbert space generated by fermion creation operators with positive (negative) weights, of J_{04} . On that factor it coincides with $\pm J_{04}$.

I believe that there is likely to be a supersymmetric version of this contraction: a q-deformed super dS algebra which contracts to two copies of the Super-Poincare algebra in the $R \sim N \rightarrow \infty$ limit. It is easy enough to write down a q-deformed dS SUSY algebra, but I have not yet found a construction of $SO_q(1,4)$, or its supersymmetric

extension, in terms of the fermion pixel operators.

To summarize, I have sketched a formalism for constructing a finite dimensional theory of global dS space which approximates the quantum field theory description of this space-time and is invariant under a quantum deformation of the dS super-group. The Hilbert space has a natural tensor split into states with positive and negative values of the static Hamiltonian of a given causal diamond. I postulated a similar split for the fermionic generators and interpreted the split super-generators as approximations to Poincare super-charges. Then, given a hypothetical set of commutation relations for these split charges in the $R \rightarrow \infty$ limit, I gave a new derivation of the scaling law for SUSY violation, which I postulated in [6]. To put these results on a firm basis we have to generalize Polychronakos oscillator construction of $U_q(N)$ quantum groups to fermions transforming in a representation of $SO_q(1, 4)$ and its supersymmetrization. Then we have to show that the generators can be split into operators in the positive and negative weight factors, which satisfy the relations 3.2 in the large R limit.

4. IR divergences in dS/CFT

From the point of view of the semi-classical approximation, the simplest way to define gauge invariant quantities for quantum gravity in asymptotically dS space is to define a path integral with boundary conditions on \mathcal{I}_\pm . This is formally invariant under coordinate transformations which approach the identity on the space-like boundaries of space-time.

There are two attempts to define such a formalism in the literature[7]. I will use the name dS/CFT, proposed by Strominger, to refer to both of them. There has been some confusion about the relation between these approaches. The confusion was cleared up in work of Maldacena[8]¹⁸. Maldacena suggests that the gauge invariant correlation functions are obtained by first computing the Hartle-Hawking wave function of the universe. This is the Euclidean path integral of a field theory which includes gravity, on a manifold with the topology of a d dimensional hemi-sphere. In the semi-classical approximation, the geometry is that of a hemi-spherical cap, with maximum polar angle θ_M . The wave functional of the universe is a functional of the boundary values of fields on this Euclidean manifold.

A baby version of this procedure is the quantization of strings. By analogy with that example, one chooses a gauge which is covariant under the infinitesimal rotation

¹⁸For the most part, Maldacena's work is not published, though hints of the formalism can be found in the reference quoted. I would like to thank J. Maldacena for explaining his approach to this problem.

group of the sphere. In Maldacena's approach, the dS/CFT correlation functions are defined by writing

$$\ln \Psi[\phi] = \sum \int \phi(\Omega_1) \dots \phi(\Omega_n) G(\theta_M, \Omega_1 \dots \Omega_n). \quad (4.1)$$

ϕ is a shorthand for all of the fields in the theory, including $h_{\mu\nu}$, the gauge fixed fluctuation of the geometry. Now, analytically continue $\theta_M \rightarrow \frac{\pi}{2} + it$ and take $t \rightarrow \infty$. If the limiting correlation functions exist, they will be covariant under the conformal group of the $d - 1$ sphere. The conformally invariant data contained in these limiting correlation functions (which would be anomalous dimensions and OPE coefficients, if the correlation functions defined a standard type of CFT) are the gauge invariant observables of quantum gravity in asymptotically dS space.

If we believe that quantum gravity in such a space-time has a finite number of states, this formalism must not be correct outside of perturbation theory. Rather, it should result from an illegitimate approximation to a finite dimensional theory, in the limit of large Hilbert space dimension. I propose that a sign of this would be the appearance of IR divergences in loop calculations of the gauge invariant data provided by the dS/CFT prescription. According to the prescription of [11], the cosmological constant itself should provide an IR regulator. IR divergences should be reinterpreted as corrections to the classical formulae for the c.c. dependence of gauge invariant quantities. In particular, one can imagine that the formula for the gravitino mass is corrected in the manner conjectured in [6].

The literature on IR divergences in dS space is large[25], but it suffers from divergences of opinion, which stem from the absence of an agreed upon set of gauge invariant quantities. The proposal here is that dS/CFT provides us with a definition of such quantities, and that IR divergences in the dimensions of boundary fields signal that the classical formulae determining the cosmological dependence of bulk masses are incorrect.

So far only a warm up calculation has been done, which might indicate that this is so. Lorenzo Mannelli, Willy Fischler, and I[12] have done one loop calculations of boundary correlators in dS space, in theories with massless minimally coupled scalars, ϕ . We found a divergence of the anomalous dimension of boundary fields dual to massive bulk fields that had soft couplings to ϕ . We are currently trying to sort out the gauge fixing subtleties of an analogous calculation in gravitational theories. The physical components of the graviton propagator have the same logarithmic behavior as a minimally coupled scalar, but one must check that the divergences do not cancel in the gauge invariant boundary dimensions.

It is conceivable that one might get the “right” formula for the dependence of the gravitino mass on the c.c. by summing up IR divergences in dS/CFT. This would be complementary to the algebraic approach of the last section. However, it is not at all clear that this is possible. I am fairly confident that purely field theoretic calculation will show that the field theory approximation is (contrary to popular opinion) not under control, but not at all confident that one can use these methods to get the right answer for quantum gravity.

5. Phenomenology of CSB

The earlier sections of this paper outlined my attempts to construct a quantum theory of a stable dS space and show that the scaling relation $m_{3/2} \sim \Lambda^{1/4}$ follows from it. Here, I want to anticipate the eventual success of those attempts, and sketch the phenomenology that results. I have made several attempts to guess the right phenomenology and will only discuss the latest, which I believe is the most successful.

There are several general features which must be shared by any phenomenological model which is derivable from CSB:

- 1.The model should have, at the level of the effective Lagrangian, a dS vacuum state, and it must contain a parameter which allows one to tune the positive cosmological constant to any value.
- 2.As $\Lambda \rightarrow 0$ it must become Super-Poincare invariant. We call the $\Lambda = 0$ Lagrangian, the *limiting theory*.
- 3.The dS vacuum state of the low energy field theory model should be absolutely stable.

These conditions can only be realized in four dimensions, with $N = 1$ SUSY, and the limiting theory must be SUSic, and invariant under a discrete ¹⁹ R symmetry which guarantees the vanishing of the superpotential. In addition, since we want to be able to turn on SUSY breaking of arbitrarily small magnitude, by turning on Λ , the limiting theory must have a massless field, which becomes the longitudinal component of the gravitino when $\Lambda \neq 0$. Since the SUSY breaking can be made arbitrarily small, this field must live in a linear supermultiplet. In the model I will discuss here, it is a chiral superfield G .

To avoid the possibility of decay of the dS minimum to a non-positive vacuum energy region of field space at large $|G|$, I will assume that the complex field G lives in

¹⁹since string theory does not have exact continuous global internal symmetries

a compact space. The size of this space might be of order M_P , or M_U , the unification scale. This assumption will not be terribly important for practical purposes, since phenomenology will suggest a much smaller range of variation of G .

I will assume that G has R charge zero, and in addition that it is charged under a discrete ordinary symmetry \mathcal{F} . G^a is the lowest order \mathcal{F} invariant holomorphic function of G .

The effective Lagrangian for the coupling of G to the standard model has the form

$$\mathcal{L} = \int d^4\theta \ G \bar{G} \ K(G/M_1, \bar{G}/M_1, S_{SM}/M_1, \bar{S}_{SM}/M_1) \quad (5.1)$$

$$+ \int d^2\theta \ \sum f_i [(G/M_1)^a] \ \text{tr}(W_\alpha^{(i)})^2 + g_\mu G H_u H_d + \Lambda^{1/4} M_P^2 f(G/M_P) + c.c. \quad (5.2)$$

Here S_{SM} stands for a generic chiral superfield in the MSSM. The rest of the standard model is standard. In particular, the small parameters in the Yukawa couplings of Higgs, quarks and leptons, are assumed to have been explained by physics (the Froggatt-Nielsen mechanism?) at the unification scale. Thus, the quark and lepton masses and mixing angles are assumed to be determined, to a good approximation by supersymmetric physics in the limiting model.

The last term in the Lagrangian is the only one which breaks the discrete R symmetry. It arises from interactions with states on the dS horizon. We will see that it leads to spontaneous SUSY breaking. The scaling of the coefficient of this term has been determined to agree with the estimate of [13] for $m_{3/2}$. Since the cosmological constant is a high energy input, one tunes the dimensionless parameters in f , in order to set it equal to its high energy input value. These two statements are the primary input of CSB ideas to the effective Lagrangian.

R symmetry charges are chosen[14] so that they forbid all dimension 4 and 5 operators that violate baryon and lepton number, apart from the dimension 5 operator that gives neutrino masses.

The Kahler potential and gauge coupling function, contain irrelevant terms scaled by the mass M_1 . We will assume that this is a scale generated by as yet undetermined strongly coupled theory, starting from a weakly coupled effective Lagrangian at the unification scale. It satisfies $M_1 \ll M_U$. The cross couplings between G and standard model fields, in the Kahler potential, are generated by a combination of this new dynamics, as well as the standard model couplings, and g_μ , at scales above M_1 .

The dynamics at scale M_1 arises from a new set of low energy degrees of freedom, which must be described by some four dimensional effective field theory below the unification scale. We denote this theory by \mathcal{G} . In order to generate the couplings to the standard model, which we have included in K and f_i , we must also have relevant or

marginal²⁰ couplings of standard model fields to these new degrees of freedom. These are assumed to consist of standard model gauge interactions and a coupling $g\mathcal{G}GO_R$, where O_R is a dimension two operator in the \mathcal{G} theory at the unification scale. As an example of such a model, we could take \mathcal{G} to be a new SUSic gauge theory, with chiral fields F_i which transform under both the \mathcal{G} group and the standard model. O_R would be a gauge invariant bilinear $c_{ij}F_iF_j$. Unfortunately, I have not yet been able to find an example of a theory whose dynamics will generate the effective Lagrangian 5.1. I emphasize that the calculation of the Kahler potential and gauge coupling function can be done, to a good approximation, in the limiting model.

The potential for G is approximately $[V(G/M_1, \bar{G}/M_1)|f'(0)|^2 - 3|f(0)|^2]\Lambda^{1/2}M_P^2$, where $V = (\partial_G\partial_{\bar{G}}[G\bar{G}K])^{-1}$. A minimum at a value of $|G| \sim M_1$ breaks SUSY, with a gravitino mass in accord (by design) with the CSB formula. $|f(0)|$ is fine tuned to guarantee that the low energy cosmological constant is Λ rather than something of order $\Lambda^{1/2}M_P^2$. From the point of view of CSB this is the right thing to do. Λ is a high energy input even though it controls the large scale structure of space-time²¹ and its value cannot be understood from a local effective field theory, nor modified by renormalization. Its value in the effective field theory *must* be tuned to agree with the fundamental definition of the theory.

The couplings of G to standard model fields now give rise to soft SUSY breaking parameters for the SSM. Assuming the canonical estimate $f_i \sim \frac{\alpha_i}{\pi}$, we get gaugino masses of order

$$m_{1/2}^i \sim \frac{\alpha_i}{\pi}\Lambda^{1/4}\frac{M_P}{M_1} \quad (5.3)$$

The experimental bound on the wino mass now implies that $M_1 \leq 1$ TeV which is uncomfortably low. Pure numerical factors could be important here. For example, if we use the reduced Planck mass in these estimates, the bound on M_1 is reduced by a factor of $\sqrt{8\pi}$, and the model is definitely in contradiction with experiment. On the other hand, we do not have a sharp estimate for either f_i or the horizon induced superpotential, which determines F_G . Either of these could raise the estimate of M_1 .

I should emphasize that, as an exercise in pure math, we can imagine doing the calculation of the effective Lagrangian for very small values of Λ . The estimate of M_1 we just made, suggests that this approximation may be inappropriate (or borderline)

²⁰possibly marginally irrelevant

²¹Indeed, *precisely because* it controls the large scale structure of space-time it also controls the spectrum of the highest (Poincare) energies (black hole states) seen by a local observer. This is the UV/IR correspondence. The reader who is confused about which energy we are talking about should review the previous section.

for the real world value of Λ . That is, in order to make calculations appropriate to the real world we may need to treat more details of the dynamics of the \mathcal{G} theory than can be incorporated in an effective Lagrangian for the MSSM coupled to G .

Terms in K of the form $G\bar{G}|S|^2$ where S is some SSM chiral superfield, will contribute to squark, slepton and Higgs boson mass terms when we insert F_G . The contributions to squark and slepton squared masses will be of order $(\frac{\alpha_i}{\pi})^2 \frac{|F_G|^2}{M_1^2}$ and come from standard model gauge loops above the scale M_1 , as in gauge mediation. They will be flavor diagonal and positive. The Higgs masses will, in addition, get contributions which depend on the Yukawa couplings g_μ and $g_{\mathcal{G}}$. If these are relatively strong they will lead to a negative contribution to the squared Higgs mass. If the μ term is not too large, this leads to spontaneous breaking of electroweak symmetry. The μ term in the SSM arises through the VEV of G and has the right order of magnitude if $|G| \sim M_1 \sim \text{TeV}$.

SUSY CP violation is not problematic in this model. Indeed, if the \mathcal{G} theory has *automatic CP conservation* then the model even solves the strong CP problem of QCD[14]. Automatic CP conservation in a gauge theory means that one can rotate away all of the CP violating phases, including the \mathcal{G} gauge theory vacuum angle. In the limit $\Lambda \rightarrow 0$, the supersymmetric low energy theory has no QCD vacuum angle because of the anomalous gluino chiral symmetry. When $\Lambda \neq 0$, and if the \mathcal{G} theory is automatically CP conserving, then, apart from the CKM matrix, CP violation appears only through a phase in F_G . This infects both the gluino mass and the B term, $F_G h_u h_d$, in a correlated way. However, using the fact that the number of generations equals the number of colors, we can rotate away both of these phases without introducing θ_{QCD} . The chiral symmetry which rotates these phases away has no QCD anomaly.

The model has no cosmological moduli problem, because the $\Lambda \rightarrow 0$ limit must be an isolated $N = 1$ vacuum, apart from the compact G field. Similarly it does not have a cosmological gravitino problem. However, because the gravitino is very light, and relatively strongly coupled, there is no SUSY dark matter. Axion dark matter is also ruled out because the freezing of moduli leaves us with no axion candidate²². The simplest possibility for dark matter is to postulate some sort of cosmologically stable \mathcal{G} hadron $B_{\mathcal{G}}$ which is a singlet of the standard model gauge group. The stability of $B_{\mathcal{G}}$ could be due to an accidental symmetry like the baryon number symmetry of QCD. The mass of this particle would probably be in the multiple TeV range and its annihilation cross section would similarly be of order $\frac{1}{(x\text{TeV})^2}$, with x a number of order one. This gives a freeze out density within shouting distance of the correct dark matter density,

²²Actually, the real part of G could be an axion candidate, but it would be ruled out by beam dump experiments.

but the details are obviously important. Also, since the stability of this particle is due to an approximate symmetry, its relic abundance will depend on the asymmetry in its approximately conserved quantum number that is generated in the early universe. It is probably necessary to assume that physics at or above the TeV scale generates an asymmetry in $B_{\mathcal{G}}$ in order to get the right relic density of these dark matter candidates

To summarize, this low energy effective description gives a model of TeV scale particle physics which is in rough accord with all data, and avoids many of the problems of conventional SUSic models. It has two principal theoretical defects:

- The scales M_1 and $\Lambda^{1/8}M_P^{1/2}$ (the maximal splitting in SUSY multiplets) are coincidentally close to each other. One should remember however that we view Λ as a variable input to the theory, whereas M_1 is a parameter which is calculable²³, and has a finite $\Lambda \rightarrow 0$ limit. Thus, it does not vary very much as $\Lambda \rightarrow 0$. Assume further that the amplitude of primordial fluctuations at horizon crossing, Q is similarly independent of Λ . Then Weinberg’s galaxy formation bound[24] bounds Λ from above by a number that is determined by M_1 (the dark matter density at the beginning of the matter dominated era). As $\Lambda \rightarrow 0$ supermultiplets become degenerate, and the gravitino becomes lighter and more strongly coupled to the standard model. At a certain point, nuclei become unstable to decay into a Bose condensate of snucleons. This puts a lower bound on Λ of order 10^{-22} times its “actual” value. An even tighter lower bound comes from requiring that electroweak symmetry breaking occur. I am not sure what the tightest lower bound on Λ is. Indeed, the definition of “tightest” depends on what facts about the real world one is willing to put in as input. In CSB Λ is an input parameter to be constrained by data. Since it is inversely proportional to the logarithm of the number of states in dS quantum gravity, *a priori* estimates of the probability distribution for small Λ will depend on a “metaphysical” model, which describes many possible universes, only one of which will ever be observed (by us). One such model was described at the end of [32]. It favors large values of Λ . If the *a priori* distribution for Λ favors larger values, then the existence of galaxies predicts the coincidence between M_1 and the largest scale of cosmological SUSY breaking. I emphasize that this conclusion depends on the extra assumption that the $\Lambda \rightarrow 0$ model is unique²⁴.

²³I am assuming here that the limiting theory is unique or that there are a very small number of possibilities. See the next section for more discussion of this point.

²⁴Or that there is a small class of possibilities, one member of which describes the universe we live in, while the other members do not have galaxies (perhaps because they have no cosmologically stable massive particles).

- The most serious defect of our model is that I have not yet found a candidate for the \mathcal{G} theory. That field theory must generate the dynamical scale M_1 without giving G a mass of order M_1 . It must preserve SUSY and the discrete symmetries, R and \mathcal{F} . It must not lead to a superpotential for G , and it must not predict extra massless degrees of freedom which are inconsistent with experiment. Discovering the \mathcal{G} theory is the most serious unfinished task in phenomenological CSB. Its dynamics determines many of the detailed predictions of our low energy effective Lagrangian.

6. Isolated Poincare invariant models of quantum gravity with four supercharges

A key ingredient in our arguments is the notion of the limiting model, the super-Poincare invariant S-matrix which arises in the limit $\Lambda \rightarrow 0$. I have argued that this has to be an isolated model, meaning that it has a compact moduli space. We do not have any examples of a model of this type coming from string theory.

The reason for this is not hard to find. In $N = 1$ SUGRA, supersymmetry is generic, but super-Poincare invariance is not. The equations for a supersymmetric model are $D_i W = 0$. This is the same number of equations as unknowns. The addition of D- terms does not change this result. Super-Poincare invariance requires $W = 0$ in addition.

There are two general strategies for finding Super-Poincare invariant vacua. The first, due to Witten[33] relies explicitly on a non-compact moduli space of approximate vacua. Let S be the non-compact direction. Assume further that the asymptotic range of S is a strip with periodic boundary conditions, like the moduli space of complex structures of a two torus. Then the superpotential has the form $W = \sum_{n=0}^{\infty} a_n e^{inS}$. Often, this is precisely the form of a BPS instanton sum: *e.g.* the imaginary part of S is the volume of some cycle, and the instanton is a brane wrapped on this cycle. If one can do the instanton calculation, and show that $a_n = 0$ for all n , then one has established the existence of a moduli space of $N = 1$ vacua.

The second strategy, first studied in [15] invokes an R symmetry, which may be discrete. In units where the superspace coordinates have charge 1, we require that there be no fields of R charge 2, and at least one field (again call it S) of R charge 0. Then the range of S is an $N = 1$ moduli space. In this case, there is no apparent barrier to assuming that the moduli space is compact. Perturbative string theory or low energy SUGRA approximations cannot find such points. Those methods rely on an approximate non-compact moduli space, and an expansion around infinity in that

moduli space. If, using these approximations, we find a vanishing superpotential on some submanifold of the moduli space, to all orders in that expansion, (and an argument that $W = 0$ non-perturbatively) then we have constructed a non-compact moduli space of $N = 1$ theories, rather than an isolated one²⁵.

Dine[35] has long argued that discrete moduli spaces should exist as points of enhanced duality symmetry. For purposes of CSB (at least with the low energy Lagrangian we have studied in this section) we actually need a smooth compact moduli space, the target space of the Goldstino field, G . It seems clear that if such moduli spaces exist, they are few and far between. This makes it difficult to find them, but suggests that the predictions they lead to will be pretty unique.

There are two possible approaches to finding compact moduli spaces. The equations $W = D_i W = 0$ for a section of a line bundle over a Kahler manifold, are topological, in the sense that they are independent of the metric on the manifold. Perhaps there is some generalization of topological string theory (or rather a non-perturbative completion of it) which finds solutions of these equations for compact moduli spaces of super-Poincare invariant models of 4 dimensional quantum gravity.

A more promising approach is to find non-perturbative dynamical equations for super-Poincare invariant models of quantum gravity. At the moment, our only examples of this come from Matrix-Theory and this only works for a subclass of models, with at least 16 supercharges²⁶. A more covariant arena for such models is a formulation on null infinity. Such a formulation cannot be a standard Hamiltonian quantum mechanics, with one of the coordinates of null infinity playing the role of time. We have to find an alternative set of dynamical equations for the S-matrix²⁷.

Many years ago, S-matrix theorists hoped that the principles of analyticity, crossing symmetry, Lorentz invariance and unitarity, might completely specify the S-matrix. Mechanically the idea is (thinking perturbatively) that the unitarity equation for the T-matrix

$$i(T - T^\dagger) = TT^\dagger \tag{6.1}$$

would allow one to calculate the discontinuities across cuts of higher order amplitudes in terms of lower order amplitudes. Then generalized Cauchy Theorems (dispersion

²⁵Recall that the the reason for insisting on a compact moduli space, was in order to ensure that the model with non-vanishing Λ cannot decay, since decay to a zero c.c. states implies an infinite number of states. Thus, I am simply recapitulating the argument that any dS state constructed in perturbative string theory will be meta-stable.

²⁶It also requires us to take a difficult large N limit to achieve super-Poincare invariance.

²⁷In four dimensions, we also have to find a more general object, along the lines of old work by Fadeev and Kulish, to deal with the problem of infrared divergences in the gravitational S-matrix.

relations) would allow one to calculate the full amplitude, and proceed by induction. If this were really true, one might suspect that the non-perturbative solution of these equations was unique. We know of course that this program was doomed to failure, because of the existence of many consistent quantum field theories. We also know that in a standard perturbative approach, even the additional constraint of local supersymmetry does not fix the S-matrix uniquely. Order by order in the low energy expansion of supergravity, expanded about a supersymmetric Minkowski vacuum one can find consistent solutions of these constraints with an ever growing number of arbitrary constants.

The analyticity postulate of S-matrix theory was never given a clear mathematical formulation, and the rules explicitly excluded massless particles and assumed high energy behavior of amplitudes which is not consistent with what we know about gravitation. Perhaps we need to revisit these questions, in order to find the proper formulation for models with compact moduli space.

Indeed, the formulation of non-perturbative rules for determining the S-matrix of super-Poincare invariant space-time, is an outstanding problem of String Theory. We have perturbative rules in various regions of moduli space, and the Matrix-theory[34] formulation for certain regions of moduli space. The latter is non-perturbative, but not fully satisfactory because it does not manifest super-Poincare invariance before taking the (difficult) large N limit. One might imagine that some of the constraints on super-Poincare invariant S matrices would be seen in a formulation like Matrix theory, only as a failure of certain models to have a Lorentz invariant large N limit. One would also like to shed light on the question of whether there are Poincare invariant theories of quantum gravity which are not Super-Poincare invariant.

7. The relation to string theory

String theorists constantly ask what the relation of all of these ideas is to “String Theory”. In my opinion, the hidden assumption behind this question is the idea that string theory is one Hamiltonian with many superselection sectors, and I think this idea is misguided. Nonetheless, we could try to give some meaning to the question.

I think one thing that cannot be true is that a stable de Sitter space has unambiguous “gauge invariant” observables, like any of the conventional string theories we know. This follows if one believes the claim that the system has only a finite number of states, and I have discussed this point extensively above. Instead I’ve suggested an equivalence class of theories of de Sitter space where some of the observables are universal in the large R limit. These are the matrix elements of an “S-matrix” which becomes the S matrix of a super-Poincare invariant limiting theory as $R \rightarrow \infty$. The ambiguities in

appropriately constrained matrix elements at finite R , are of order $e^{-R^{3/2}}$. The phrase *appropriately constrained* in the previous sentence means: a process in asymptotically flat space, whose S-matrix is well approximated by finite time amplitudes in a region whose linear size is $\ll R$.

I would like to believe that the limiting model shares properties of the most realistic string compactifications we know, the *M-theory on G2/Heterotic on CY3/F-theory on CY4* moduli space. and that it is relatively unique²⁸.

There is a hypothetical way in which string theory could lead us to a model of dS space by calculation instead of analogy. The basic idea is to construct some sort of brane in asymptotically flat space, whose local geometry is de Sitter, and find a decoupling limit where a finite number of states associated with the brane, cease to interact with the rest of the system. The decoupling cannot be precise, because there is a gap in the energy spectrum of the dS Hamiltonian, so a low energy limit cannot be taken. However, since the gap is exponentially small for large radius, it might be possible to have approximate decoupling. The residual, exponentially small, couplings to the rest of string theory, would be examples of the imprecision in the definition of the dS Hamiltonian. Different states of the external string theory, would induce slightly different Hamiltonians on the brane. In a situation like this, it is likely that the dS brane will be unstable, but if its lifetime is longer than the tunneling times of field theoretic apparatus in dS space ($e^{R^{3/2}}$) then it would enable us to extract a dS Hamiltonian with the degree of precision measurable in dS space. It may be that the results of [36] should be viewed as attempts to implement this decoupling strategy.

8. Conclusions

Cosmological SUSY Breaking is a work in progress. While I believe that I have uncovered tantalizing hints of an elegant formulation of it, they are incomplete. The first new result in the present paper was a clarification of the relation between Poincare and static Hamiltonians in the static gauge. This led to a successful prediction of the relation between black hole mass and entropy for dS black holes. I review the construction²⁹[17] of a quantum model of the static gauge Poincare generator, which reproduced the properties of Schwarzschild de Sitter black holes. I then proposed a new algebraic program for constructing a theory of stable dS space in global gauge, and outlined a possible algebraic derivation of the CSB scaling relation. A final new result

²⁸Sufficiently unique that one can distinguish the correct model of our world, from the class of all mathematically consistent theories, by relatively simple criteria like the existence of galaxies.

²⁹in collaboration with B. Fiol,

was an explanation of the coincidence between the dynamical scale M_1 and the CSB scale $\Lambda^{1/8} M_P^{1/2}$, in the low energy phenomenology that follows from CSB.

There are, as I see it, three or four lines of attack on the proposal of CSB:

- A direct assault on the description of dS in global gauge. Here one must construct the q-deformed super dS generators and verify that they can be decomposed into approximate Poincare super-generators for North and South causal diamonds. The crucial results will be to show that the bound on the spectrum of the Poincare Hamiltonian, P^0 , scales like $N^{3/2}$ and that the leading term in the commutator between P^0 and the restriction of the SUSY generator to one horizon, scales like the full SUSY generator.
- Continued investigation of the low energy phenomenology suggested by CSB. The crucial step here is to find an appropriate strongly coupled theory, \mathcal{G} .
- Construction of a Non-Perturbative algorithm for Super-Poincare invariant S-matrices in quantum gravity, in order to investigate the existence of isolated models in four dimensions with minimal SUSY.
- An attempt to derive the CSB scaling relation by summing up infrared divergences in perturbative dS/CFT. I am somewhat skeptical that this can give the correct result. The dS/CFT calculations may demonstrate the existence of large corrections to the naive classical relation between the gravitino mass and the c.c., but they may not be sufficient to extract the true behavior in quantum gravity.

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